values for $n = 16(1)50$ and $q = 1(1)100$.

The characteristic values b_{2n}, b_{2n+1} (associated with the odd periodic solution) are tabulated on pp. 112–150 for $q = 0.1(0.1)100$ and $n = 0(1)m$, where m increases from 1 to 9 for b_{2n} and from 0 to 8 for b_{2n+1}, with increasing values of q.

Immediate comparison of these tables is possible with the considerably abridged 8D table (Table 20.1) in the NBS *Handbook* [1]. Such a comparison has revealed to this reviewer that the Russian values were simply chopped at seven significant figures, thereby resulting in last-figure tabular errors approaching a unit. It might be pointed out here that greater precision (over the more restricted ranges of $n \leqq 7$ and $q \leqq 25$) can also be obtained from another NBS publication [2], in conjunction with the relations $a_n = be_n - 2q$, $b_n = bo_{n+1} - 2q$, and $s = 4q$.

An introduction to the present tables describes their contents and preparation and includes three illustrative examples of the application of appropriate interpolative procedures. The appended list of seven references does not include any of the pertinent NBS publications, which contain extensive bibliographies relating to Mathieu functions.

Despite such defects, these tables contain much new numerical information, constituting a valuable addition to that available in previous tables of the characteristic values of the Mathieu equation.

J. W. W.

1. MILTON ABRAMOWITZ & IRENE STEGUN, Editors, *Handbook of Mathematicial Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964.

2. *Tables Relating to Mathieu Functions*, National Bureau of Standards, Applied Mathematics Series, No. 59, U. S. Government Printing Office, Washington, D. C., 1967.

10 [7].—YUDELL L. LUKE, *The Special Functions and Their Approximations*, Academic Press, New York, 1969, vol. I, xx + 349 pp., vol. II, xx + 485 pp., 23 cm. Price $19.50 per volume.

The special functions of mathematical physics are, simply, those functions which arise most frequently in the classical problems of applied mathematics and physics. Setting aside the elementary functions (logarithmic, exponential, circular and hyperbolic), there remain a number of well-known, named functions which are of very wide application. They are usually functions of more than one variable, satisfying fairly simple linear differential equations from which many basic properties may be derived. Such properties have been explored by many authors; Y. L. Luke's contribution has been the development of a unified basis for these special functions with special attention to methods for their computation. They are all treated as special cases of the hypergeometric functions.

Treatment of the Gaussian hypergeometric function ${}_2F_1$ embraces the Legendre functions, the incomplete beta function, the complete elliptic functions of the first and second kinds, and the familiar systems of orthogonal polynomials. The confluent hypergeometric function ${}_1F_1$ includes the Bessel functions and their relatives, and the incomplete gamma function. Connections with a still wider class of functions are demonstrated in a discussion of Meijer's G-function, a generalization of the hypergeometric function. It is shown that each of the most commonly used functions of analysis is a special case of the G-function, which also has relevance to many functions which are not hypergeometric in character.

Volume I contains eight chapters and an extensive bibliography. Chapter 1 is a very brief introduction to asymptotic expansions and Watson's lemma, and is followed by chapters headed: 2. The gamma function and related functions, 3. Hypergeometric functions, 4. Confluent hypergeometric functions, 5. The generalized hypergeometric function and the G-function, 6. Identification of the ${}_pF_q$ and G-functions with the special functions of mathematical physics, 7. Asymptotic expansions of ${}_pF_q$ for large parameters and 8. Orthogonal polynomials.

Volume II includes: 9. Expansions of generalized hypergeometric functions in series of functions of the same kind, 10. The τ-method, 11. Polynomial and rational approximations to generalized hypergeometric functions, 12. Recursion formulas for polynomials which occur in infinite series and rational approximations to generalized hypergeometric functions, 13. Polynomial and rational approximations for $E(z) = {}_2F_1(1, \sigma; \rho + 1; -1/z)$, 14. Polynomial and rational approximations for the incomplete gamma function, 15. Trapezoidal rule integration formulas, 16. Applications and 17. Tables of coefficients. These chapters are followed by a copy of the bibliography appearing in Volume I.

There is a vast amount of information contained in this work, which explores thoroughly the interconnections between the functions considered. Asymptotic series for large absolute values of the argument are developed in all appropriate cases, and series in terms of orthogonal polynomials, notably Chebyshev polynomials of the first kind, are also explored. Most of this groundwork is laid in the first volume, while in the second the emphasis is on computation, with discussion of general methods of approximation and treatment of particular numerical approximations for the special functions.

The author says that his book is primarily intended as a reference tool. As such, it can be welcomed as a significant addition to the literature. It is evident that the tables of Chapter 17 have been constructed with considerable care, and they will undoubtedly be used widely and often. The comprehensive listing of the various relations between the special functions and the hypergeometric functions, and of the important asymptotic series will also be consulted frequently. The background to the methods of approximation is a valuable companion to the approximations themselves.

There is also, however, a suggestion that the book might be used as a text for undergraduate or graduate courses in special functions. Such use should be implemented with caution. Most of the mathematical development is very terse, and much of it sketchy. Moreover, it is not uniformly reliable. There is, for instance, a Theorem 5 in Chapter 8 (Section 8.5.3) which states that "If $Q_n(x)$ is the best polynomial approximation to $f(x)$ of degree n, then $f(x) - Q_n(x) = \omega T_{n+1}(x)$,

$$\omega = \max_{a \le x \le b} |f(x) - Q_n(x)|."$$

One must, of course, expect a few slips in a work of such magnitude, but it is difficult here even to guess what was intended. Fortunately (though perhaps confusingly for the student), this theorem is contradicted by the next sentence.

The blemishes will be revealed, and no doubt removed, in time. They cannot obscure the value of a reference book which no practical mathematician can afford to overlook.

C. W. CLENSHAW

Mathematics Division
National Physics Laboratory
Teddington, Middlesex
England

11 [7].—KENNETH L. MILLER, PAUL MOLMUD & WILLIAM C. MEECHAM, *Tables of the Functions*

$$G(x + iy) = \int_0^\infty \frac{e^{-t^2}}{t - (x + iy)}\, dt \quad and \quad F(x + iy) = \int_0^\infty \frac{t^4 e^{-t^2}}{t - (x + iy)}\, dt,$$

Report 6121-6249-RU-000, Space Technology Laboratories, Redondo Beach, California, 8 February 1963, 17 pp. + tables (consisting of 102 unnumbered pp.), deposited in the UMT file.

The tables in this report, only recently submitted for deposit in the UMT file, consist of 6S decimal values in floating-point form of the infinite integrals identified in the title, for the ranges $x = -10(0.2)10$ and $y = 0(0.2)10$. As the authors note, values of these integrals in the lower half-plane are immediately obtainable by taking the complex conjugates of the corresponding values in the upper half-plane.

In the introduction to these tables it is shown that $F(\zeta)$ is expressible in terms of $G(\zeta)$, where $\zeta = x + iy$; consequently, only the properties of $G(\zeta)$ are discussed in detail. In particular, the relation of this function to the complex error function and the complex exponential integral is set forth.

Computation of the tables was performed on an IBM 7090 system, using single-precision floating-point arithmetic. Except for the range $x \geqq 0$, $0 \leqq y < 0.8$, the function $G(\zeta)$ was evaluated from its continued-fraction representation, derived by the quotient-difference algorithm; in the remaining range of the tabular arguments the function was evaluated by numerical integration of the first-order linear differential equation that it satisfies. The corresponding values of $F(\zeta)$ were then deduced by means of the stated identity relating the two functions.

Asymptotic series are presented for the calculation of these integrals outside the range of the tables. Also, calculation of intermediate values by bilinear interpolation and by Taylor's series is briefly discussed.

The bibliography, consisting of nine references, omits a pertinent paper of Goodwin & Staton [1], which contains a 4D table of $G(\zeta)$ for $-x = 0(0.02)3(0.1)10$, $y = 0$.

In addition to their immediate use in the theoretical determination of the alternating current electrical conductivity of weakly ionized gases, these tables can also be applied, as the authors note, in the theory of plasma oscillations and also in the theory of the thermoelectric properties of metals and semiconductors.

J. W. W.

1. E. T. GOODWIN & J. STATON, "Table of $\int_0^\infty e^{-u^2}/(u + x)\, du$," *Quart. J. Mech. Appl. Math.*, v. 1, 1948, pp. 319–326.